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SOLUTIONS.

129 (Average and probabilities) [**1902, 148; 1903, 81**]. **Proposed by J. K. ELLWOOD, Pittsburg, Pa.**

A and B play with two dice, A throwing. If he throws 7 or 11, he wins; if he throws 3, or two aces, or two sixes, B wins. But if he throws 4, 5, 6, 8, 9, or 10, he continues throwing to duplicate this throw, in which event he wins; if in throwing, however, he throws 7, B wins. What is the expectancy of each? [This is the regulation "crap" game, B being the banker.]

155 (Average and probabilities) [**1905, 76**]. **Proposed by E. B. WILSON, Yale University.**

The game of craps is played with two dice. If the player throws 7 or 11 on the first throw he wins. If he throws 12, 2, or 3 he loses. If the player throws any other number, that is to say, 4, 5, 6, 8, 9, 10, he is obliged to continue throwing until he throws that number again or until he throws 7. If he succeeds in throwing his first throw before he does 7, he wins—otherwise he loses. Required the odds against him. (Note that he can continue throwing indefinitely without getting either his original throw or the 7).

NOTE BY R. C. ARCHIBALD, Brown University.

The answers to these problems may be found in the article by Mr. Bancroft H. Brown ("Probabilities in the game of 'shooting craps'") in this MONTHLY, *1919*, 351–352; see also *1920*, 166–167.

2752 [1919, 72]. Proposed by the late R. E. MOORE.

Test for convergence, the series $\sum\limits_{n=1}^{\infty} a_n$, in which

$$a_n = \left[\frac{1\cdot 3\cdot 5 \cdots (2n-1)}{2\cdot 4\cdot 6 \cdots 2n}\right]^2.$$

I. SOLUTION BY P. J. DA CUNHA, University of Lisbon.

On sait que lorsque le rapport a_n/a_{n+1} est développable suivant les puissances entières de $1/n$, il est très facile de décider, dans tous les cas, s'il y a ou non convergence. En effet, si l' on pose, en s'arrêtant aux termes du second ordre,

$$\frac{a_n}{a_{n+1}} = \alpha + \frac{\beta}{n} + \frac{\theta_n}{n^2},$$

θ_n restant fini pour $n = \infty$, il y a:

Divergence si $\alpha < 1$ ou $\alpha = 1$ et $\beta \leqq 1$;
Convergence si $\alpha > 1$ ou $\alpha = 1$ et $\beta > 1$

(Jordan, *Cours d'Analyse*, troisième édition, tome 1, page 313.)

Cela posé, en appliquant la règle à la série donnée, nous avons

$$\frac{a_n}{a_{n+1}} = \left(\frac{2n+2}{2n+1}\right)^2 = \frac{4n^2+8n+4}{4n^2+4n+1} = \frac{1+\dfrac{2}{n}+\dfrac{1}{n^2}}{1+\dfrac{1}{n}+\dfrac{1}{4n^2}}$$

ou, finalement

$$\frac{a_n}{a_{n+1}} = 1 + \frac{1}{n} + \frac{\theta_n}{n^2}.$$

Comme nous trouvons $\alpha = 1$, $\beta = 1$, la serie considerée est divergente.

II. SOLUTION BY OTTO DUNKEL, Washington University.

A proof of the same nature as that of Professor Cunha but requiring only elementary facts is as follows.

Omit the first factor $(\frac{1}{2})^2$ of each term and consider the series whose general term is

$$b_n = 4a_n = \left(\frac{3}{4}\right)^2\left(\frac{5}{6}\right)^2 \cdots \left(\frac{2n-3}{2n-2}\right)^2\left(\frac{2n-1}{2n}\right)^2.$$

Since

$$\left(\frac{2n-1}{2n}\right)^2 = \left(1-\frac{1}{2n}\right)^2 = 1 - \frac{1}{n} + \frac{1}{4n^2} > \frac{n-1}{n},$$

we have, by multiplying all such inequalities from $n = 2$ to $n = n$,

$$b_n > \frac{1}{n}$$

and therefore the b series is divergent and hence also the a series.

Also solved by E. H. CLARKE, R. A. JOHNSON, H. L. OLSON, ARTHUR PELLETIER, and the Proposer.

2758 [1919, 124]. Proposed by LEONARD RICHARDSON, University of British Columbia.

Prove that, if r be a positive integer,

$$\int_0^{\pi/2} \frac{\sin(2r+1)\psi}{\sin\psi}\,d\psi = \frac{\pi}{2}.$$

and

$$\int_0^{\pi/2} \frac{\sin 2r\psi}{\sin\psi}\,d\psi = 2\left\{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^{r-1}}{2r-1}\right\}.$$

I. SOLUTION BY ELIJAH SWIFT, University of Vermont.

We have the identity

$$\sin n\psi - \sin(n-2)\psi = 2\sin\psi\cos(n-1)\psi,$$

or

$$\sin n\psi = 2\sin\psi\cos(n-1)\psi + \sin(n-2)\psi.$$

This yields us an easy reduction formula for the two integrals. For the second,

$$\int_0^{\pi/2} \frac{\sin 2r\psi}{\sin\psi}\,d\psi = 2\int_0^{\pi/2} \cos(2r-1)\psi\cdot d\psi + \int_0^{\pi/2} \frac{\sin(2r-2)\psi}{\sin\psi}\,d\psi$$

$$= 2\frac{(-1)^{r-1}}{2r-1} + \int_0^{\pi/2} \frac{\sin(2r-2)\psi}{\sin\psi}\,d\psi.$$

Applying this r times, we reach the indicated result.

For the first integral,

$$\int_0^{\pi/2} \frac{\sin(2r+1)\psi}{\sin\psi}\,d\psi = 2\int^{\pi/2} \cos 2r\psi\cdot d\psi + \int_0^{\pi/2} \frac{\sin(2r-1)\psi}{\sin\psi}\,d\psi.$$

The first of these is 0. Repeating this process r times, the required integral

$$= \int_0^{\pi/2} \frac{\sin\psi}{\sin\psi}\cdot d\psi = \frac{\pi}{2}.$$

Solved similarly by A. M. HARDING and C. C. YEN.

II. SOLUTION BY R. D. BOHANNAN, Ohio State University.

Let $\cos\psi + i\sin\psi = z$ and $\cos\psi - i\sin\psi = 1/z$. Then, for the first integral, we have

$$\frac{\sin(2r+1)\psi}{\sin\psi}\,d\psi = \frac{z^{2r+1} - \dfrac{1}{z^{2r+1}}}{z - \dfrac{1}{z}}\frac{dz}{iz},$$

$$= \left\{z^{2r} + z^{2r-2} + \cdots + z^2 + 1 + \frac{1}{z^2} + \cdots + \frac{1}{z^{2r}}\right\}\frac{dz}{iz}.$$

The first and last terms, after integration, give

$$\frac{1}{2ri}\left(z^{2r} - \frac{1}{z^{2r}}\right) \quad \text{or,} \quad \frac{1}{r}\sin 2r\psi\Big]_0^{\pi/2},$$

which is zero; likewise, all other pairs, equidistant from the ends. The middle term gives $(\log z)/i$ or ψ, since $z = e^{i\psi}$, and with the given limits, this reduces to $\pi/2$.